RELOG: MULTI ECHELON COMPOSITION MODEL

SETS

Q	CENTERS
P	PLANTS
R	PRODUCTS AND MATERIALS
C_r	COMPONENTS OF PRODUCT r
$E \subseteq (Q \cup P) \times P \times R$	TRANSPORTATION EDGES
$T = \{1, \dots, t^{MAX}\}$	TIME PERIODS
S	SET OF EMISSIONS
R_p^{OUT}	OUTPUT PRODUCTS OF PLANT p
R_p^{W}	INPUT PRODUCT OF PLANT p

DATA

COLLECTION

m_{qrct}^{INIT}	AMOUNT INITIALLY AVAILABLE
$m_{rt}^{C\text{-}DISP}$	MAXIMUM DISPOSAL ACROSS ALL COLLECTION CENTERS
$c_{qrt}^{C\text{-}DISP}$	DISPOSAL COST AT COLLECTION CENTER
c_{qrt}^{ACQ}	ACQUISITION COST
m_{qrt}^{STORE}	STORAGE CAPACITY
c_{qrt}^{STORE}	STORAGE COST

TRANSPORTATION

c_{rt}^{TR}	TRANSPORTATION COST ($/KM/TONNE)
m_{qp}^{DIST}	DISTANCE
$\alpha_{rdt}^{TR\text{-}EMISSION}$	TRANSPORTATION EMISSIONS FACTOR

PLANT

c_{pt}^{FIX}	FIXED OPERATING COST
c_{pt}^{VAR}	VARIABLE OPERATING COST
c_{pt}^{OPEN}	OPENING COST
m_{p}^{CAP}	PLANT CAPACITY
$\alpha_{p,r,c_{IN},c_{OUT}}^{MIX}$	OUTPUT FACTOR
$m_{prt}^{P\text{-}DISP}$	DISPOSAL LIMIT
$c_{prt}^{P\text{-}DISP}$	DISPOSAL COST
$\alpha_{prt}^{P\text{-}EMISSION}$	PLANT EMISSION FACTOR

EMISSIONS

$c_{mt}^{EMISSION}$	EMISSION PENALTY
$m_{mt}^{EMISSION}$	EMISSION LIMIT

DECISION VARIABLES

y_{qprct}	MATERIAL FLOW (COMPONENT)
y_{qprt}^{TOTAL}	MATERIAL FLOW (PRODUCT)
$z_{qrct}^{C\text{-}DISP}$	CENTER DISPOSAL (COMPONENT)
$z_{qrt}^{C\text{-}DISP\text{-}TOTAL}$	CENTER DISPOSAL (PRODUCT)
z_{qrct}^{STORE}	CENTER STORAGE (COMPONENT)
$z_{qrt}^{STORE\text{-}TOTAL}$	CENTER STORAGE (PRODUCT)
x_{pt}^{OPEN}	IS PLANT OPEN?
x_{pqrt}^{SEND}	DOES PLANT p SEND TO q?
x_{prt}^{DISP}	DOES PLANT p DISPOSE OF r?
z_{prct}^{PROD}	AMOUNT PRODUCED
$z_{prct}^{P\text{-}DISP}$	AMOUNT DISPOSED
$z_{qprmt}^{TR\text{-}EMISSIONS}$	TRANSPORTATION EMISSIONS
$z_{pmt}^{P\text{-}EMISSIONS}$	PLANT EMISSIONS

OBJECTIVE FUNCTION

$$\min \sum_q \sum_t \sum_r z_{qrt}^{C\text{-}DISP\text{-}TOTAL} \cdot c_{qrt}^{C\text{-}DISP} \quad \text{CENTER DISPOSAL}$$

$$+ \sum_q \sum_t \sum_r \sum_{c \in C_r} m_{qrct}^{INIT} \cdot c_{qrt}^{ACQ} \quad \text{CENTER ACQUISITION}$$

$$+ \sum_q \sum_r \sum_t z_{qrt}^{STORE\text{-}TOTAL} \cdot c_{qrt}^{STORE\text{-}TOTAL} \quad \text{CENTER STORAGE}$$

$$+ \sum_{(q,p,r) \in E} \sum_{c \in C_r} \sum_t y_{qprct} \cdot m_{qp}^{DIST} \cdot c_{rt}^{TR} \quad \text{TRANSPORTATION}$$

$$+ \sum_{(q,p,r) \in E} \sum_m \sum_t z_{qprmt}^{TR\text{-}EMISSIONS} \cdot c_{mt}^{EMISSION} \quad \text{TR EMISSION}$$

$$+ \sum_p \sum_t c_{pt}^{FIX} \cdot x_{pt}^{OPEN} \quad \text{FIXED COST}$$

$$+ \sum_p \sum_t c_{pt}^{OPEN} \cdot \left(x_{pt}^{OPEN} - x_{p,t-1}^{OPEN} \right) \quad \text{OPENING COST}$$

$$+ \sum_{(q,p,r) \in E} \sum_{c \in C_r} \sum_t y_{qprct} \cdot c_{pt}^{VAR} \quad \text{VAR. OPERATING COST}$$

$$+ \sum_p \sum_r \sum_c \sum_t z_{prct}^{P\text{-}DISP} \cdot c_{prt}^{P\text{-}DISP} \quad \text{PLANT DISPOSAL}$$

$$+ \sum_p \sum_m \sum_t z_{pmt}^{P\text{-}EMISSIONS} \cdot c_{mt}^{EMISSION} \quad \text{PLANT EMISSIONS}$$

CONSTRAINTS

- MATERIAL BALANCE AT CENTERS:

$$\sum_{(p,r) \in E^+(q)} y_{qprt}^{TOTAL} + z_{qrt}^{STORE\text{-}TOTAL} = \sum_{c \in C_r} m_{qrct}^{INIT} + z_{qr,t-1}^{STORE\text{-}TOTAL} \qquad \forall q, r, t$$

- SPLIT TOTALS

$$y_{qprct} = \frac{m_{qrct}^{INIT}}{\sum_{d \in C_r} m_{qrdt}^{INIT}} \cdot y_{qprt}^{TOTAL} \qquad \forall (q,p,r) \in E, t, c : q \in Q$$

$$z_{qrct}^{C\text{-}DISP} = \frac{m_{qrct}^{INIT}}{\sum_{d \in C_r} m_{qrdt}^{INIT}} \cdot z_{qrt}^{C\text{-}DISP\text{-}TOTAL} \qquad \forall q, r, c, t$$

$$z_{qrct}^{STORE} = \frac{m_{qrct}^{INIT}}{\sum_{d \in C_r} m_{qrdt}^{INIT}} \cdot z_{qrt}^{STORE\text{-}TOTAL} \qquad \forall q, r, c, t$$

- CENTER DISPOSAL LIMIT:

$$\sum_q z_{qrt}^{C\text{-}DISP\text{-}TOTAL} \le m_{rt}^{C\text{-}DISP} \qquad \forall r, t$$

- CENTER STORAGE LIMIT:

$$z_{qrt}^{STORE\text{-}TOTAL} \le m_{qrt}^{STORE} \qquad \forall q, r, t$$

- TRANSPORTATION EMISSIONS:

$$z_{qprmt}^{TR\text{-}EMISSIONS} = m_{qp}^{DIST} \cdot \alpha_{rmt}^{TR\text{-}EMISSION} \cdot y_{qprt}^{TOTAL} \qquad \forall (q,p,r) \in E, m, t$$

- PLANT CAPACITY

$$\sum_{(q,r) \in E^-(p)} y_{qprt}^{TOTAL} \le m_p^{CAP} \cdot x_{pt}^{OPEN} \qquad \forall p, t$$

- PLANT PRODUCTION:

$$z_{prdt}^{PROD} = \sum_{(q,r) \in E^-(p)} \sum_{c \in C_r} \alpha_{prcd}^{MIX} \cdot y_{qprct} \qquad \forall p, r \in R_p^{OUT}, d \in C_r, t$$

- PLANT DISPOSAL LIMIT:

$$\sum_{c \in C_r} z_{prct}^{P\text{-}DISP} \le m_{prt}^{P\text{-}DISP} \cdot x_{prt}^{DISP} \qquad \forall p, r \in R_p^{OUT}, t$$

- PLANT EMISSIONS:

$$z_{pmt}^{P\text{-}EMISSIONS} = \sum_{(q,r) \in E^-(p)} y_{qprt}^{TOTAL} \cdot \alpha_{prmt}^{P\text{-}EMISSION} \qquad \forall p, m, t$$

- EMISSIONS LIMIT:

$$\sum_p z_{pmt}^{P\text{-}EMISSIONS} + \sum_{(q,p,r) \in E} z_{qprmt}^{TR\text{-}EMISSIONS} \le m_{mt}^{EMISSION} \qquad \forall m, t$$

- PLANT REMAINS OPEN:

$$x_{pt}^{OPEN} \ge x_{p,t-1}^{OPEN}$$

- PLANT HAS SINGLE DESTINATION:

$$\sum_{(q,\bar{r}) \in E^+(p) : r = \bar{r}} x_{pqrt}^{SEND} + x_{prt}^{DISP} \le 1 \qquad \forall p, r, t$$

- PLANT SENDING LIMIT:

$$y_{pqrt}^{TOTAL} \le m_q^{CAP} \cdot x_{pqrt}^{SEND} \qquad \forall p, (q,r) \in E^+(p), t$$

- PLANT MATERIAL BALANCE:

$$z_{prct}^{PROD} = z_{prct}^{P\text{-}DISP} + \sum_{(q,\bar{r}) \in E^+(p) : r = \bar{r}} y_{pqrct} \qquad \forall p, r, c, t$$

- TOTAL FLOW

$$y_{qprt}^{TOTAL} = \sum_{c \in C_r} y_{qprct} \qquad \forall (q,p,r) \in E, t$$